SWAMI VIVEKANANDA SCHOOL OF ENGG.
& TECH.

FC – II

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ALTERNATOR

→ ALTERNATORS → Alternators are operate on the principle of electromagnetic induction.
→ In Alternator armature is stationary and the field system is rotates.
→ When the rotor rotates the stator conductors are cut by the magnetic flux, hence they can have induced emf in it. An alternating emf is produced in the stator conductors.
(i) whose frequency depends on the number of N and S ~~pole~~ poles.
(ii) whose direction is given by Fleming's right hand rule.

Stationary Armature

Advantage → The output current can be led directly from fixed terminals on the stators on the load circuit.
(ii) It is easier to ~~s~~ insulate stationary armature winding for high ac voltages, when (may be 30KV)
(iii) The armature windings can be more easily braced to prevent any deformation.

ROTOR

① Salient pole type → It is used in low and medium speed alternater.
→ It has large number of projecting poles. Such generators are characterised by large diameters and short axial length.
→ The poles and pole shoes are laminated to minimize heating due to eddy currents.

Smooth cylindrical Type

→ It is used for steam turbine-driven alternators.
→ The rotor consists of a smooth solid forged ste[el] cylinder, having
→ such rotors are designed mostly for 2-pole turbo generators running at 3600 rpm.

DAMPER WINDINGS

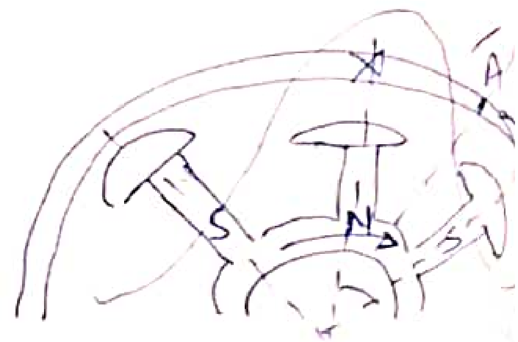
The damper bars are useful in preventing th[e] hunting. Turbogenerators usually donot have th[e] damper windings

The damper winding also used to maintain balanced 3-ϕ voltage under unbalanced load conditions.

speed and Frequency

In an alternator there exist a definite relation

M → rotational speed
f → Frequency
P → No of poles.



When the conductor is in the interpolar gap it has minimum emf induced in it, because flux density is minimum.
→ Again when it is at the centre of a s-pole, it has maximum emf induced in it, because flux densit[y] cut is maximum.

→ But the direction of emf when conductor over a N-pole is opposite to that when it is over a S-pole.
→ one cycle of emf is induced in a conductor when one pair of poles passes over it.

Since one cycle emf is produced when a pair of poles passes past a conductor the numbers of cycles of emf produced in one revolution of the rotor is equal to the number of poles pair of poles.

~~freq~~ No of cycles/revolution $= P/2$

No of revolutions/second $= N/60$

$$\text{frequency} = P/2 \times N/60 = \frac{PN}{120} \text{ Hz}$$

$$\boxed{f = \frac{PN}{120} \text{ Hz}}$$

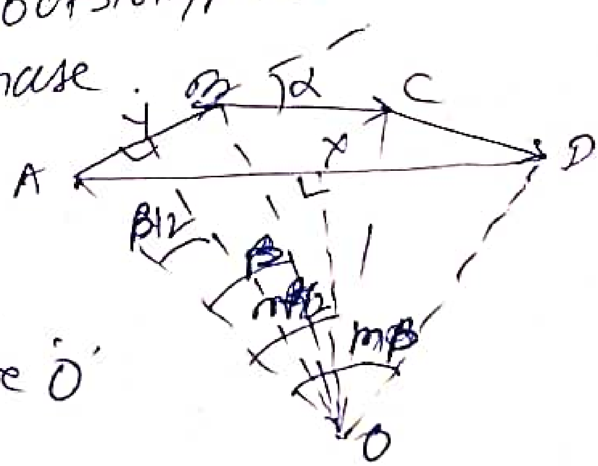
Distribution factor or winding factor

$$K_d = \frac{\text{emf with distributed winding}}{\text{emf with concentrated winding}}$$

$$= \frac{\text{phasor sum of coil emfs/phase}}{\text{arithmatic sum of coil emfs/phase}}$$

let β = slot angle $= \frac{180° \text{ electrical}}{\text{No of slots/pole}}$

m = slots/pole/phase.



eg let $m = 3$

The three coil emfs are AB, BC and CD. which is a chord of circle with centre O

$$K_d = \frac{AD}{m\,AB} = \frac{2AX}{m(2AY)} = \frac{AX}{m\,AY}$$

$$= \frac{OA \sin(m\beta/2)}{m \times OA \sin\beta/2}$$

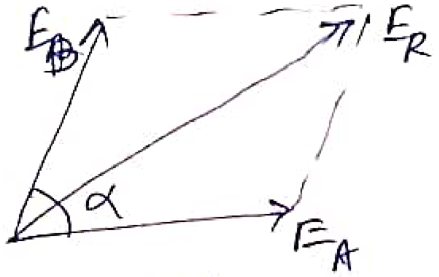
$$\boxed{K_d = \frac{\sin m\beta/2}{m \sin\beta/2}}$$

$m\beta \rightarrow$ phase spread.

Pitch factor (K_p)

$$K_p = \frac{\text{emf induced in short pitch coil}}{\text{emf induced in full pitch coil}}.$$

Expression $\rightarrow$ consider a coil AB is short pitch, an angle α.



Since $E_A = E_B$ $E_R = 2E_A \cos\alpha/2$

$$K_p = \frac{\text{emf in short pitch coil}}{\text{emf in Full pitch coil}} =$$

$$= \frac{2E_A \cos\alpha/2}{2E_A} = \cos\alpha/2$$

$$\boxed{K_p = \cos\alpha/2}$$

FACTORS AFFECTING ALTERNATOR SIZE

The efficiency of an alternator increases as its power increases.

→ Another advantages of large machines is that power output per kilogram increases.

→ As alternator size increases cooling problem become more serious.

ALTERNATOR ON LOAD

→ As ~~of~~ load on alternator is varied, the variation in terminal voltage V depends on

(i) voltage drop due to armature resistance R_a

(ii) voltage drop due to armature leakage reactance X_L

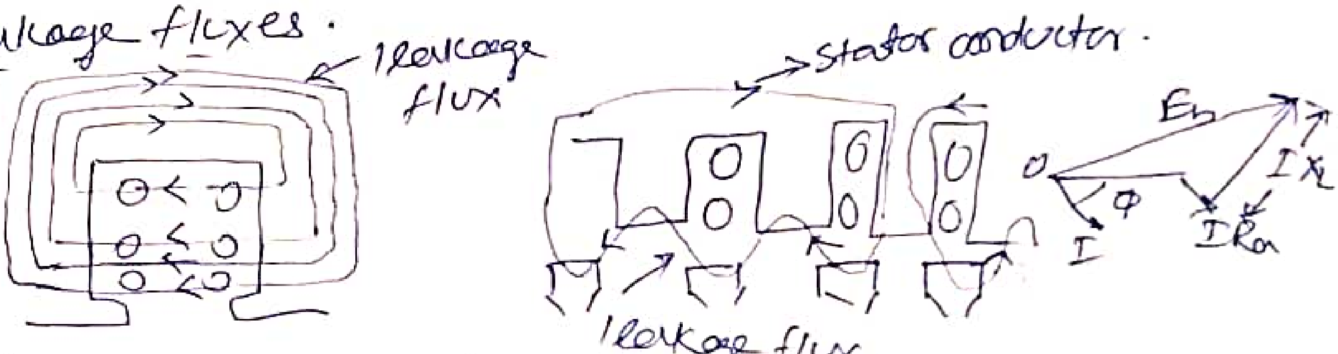
(iii) voltage drop due to armature reaction.

(a) Armature Resistance

→ The armature resistance/phase R_a causes a voltage drop/phase of IR_a is in phase with I.

(b) Armature Leakage Reactance

→ When current flows through the armature conductors fluxes are set up which donot cross the air gap, but take different paths, such fluxes are known as leakage fluxes.



→ leakage flux is dependent on I and its phase angle with terminal voltage V. This leakage flux sets up an emf which is known as reactance emf & ahead of I by 90°.

EQUATION OF INDUCED EMF

let $Z \rightarrow$ No of conductors or coil sides in series/phase
$= 2T$ where $T \rightarrow$ No of coils
(one turn or coil has two sides)

P = No of poles
f = frequency of induced emf in Hz
ϕ - flux/pole in wb

$K_d \rightarrow$ distribution factor $= \frac{\sin m\beta/2}{m \sin \beta/2}$

K_c or $K_p \rightarrow$ Pitch factor

$K_f \rightarrow$ Form factor 1.11

$N \rightarrow$ rotor rpm.

In one revolution of the rotor each stator conductor is cut by a flux of ϕP wb.

$d\phi = \phi P$ and $dt = 60/N$ second.

Avg. emf induced/conductor $= \frac{d\phi}{dt} = \frac{\phi P}{60/N} = \frac{\phi N P}{60}$

$f = \frac{PN}{120}$ or $N = \frac{120 f}{P}$

Avg. emf/conductor $= \frac{\phi P}{60} \times \frac{120 f}{P} = 2 f \phi$ volt.

If there are Z conductor in series/phase

Average emf/phase $= 2 f \phi Z$ volt $= 4 f \phi T$ v

Rms value of emf/phase $= 1.11 \times 4 f \phi T = 4.44 f \phi T$ volt.

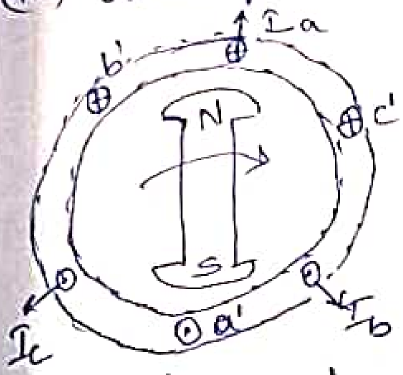
Actually available voltage/phase $= 4.44 K_c K_d f \phi T$ volt.

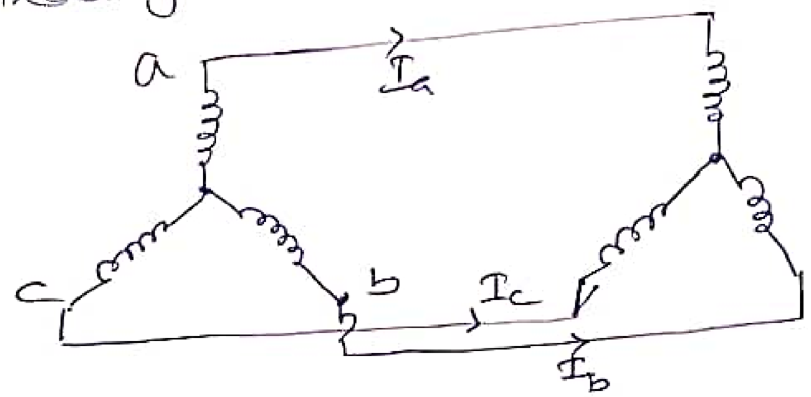
$E = V + I(R + jX_L)$

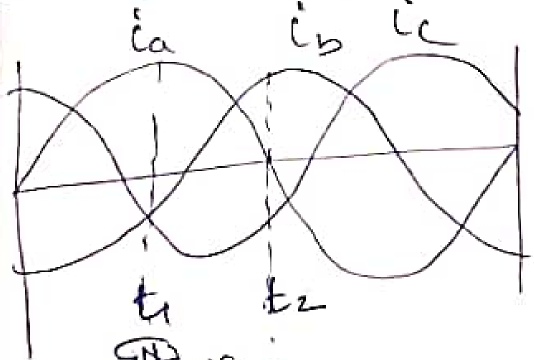
4(c) Armature Reaction

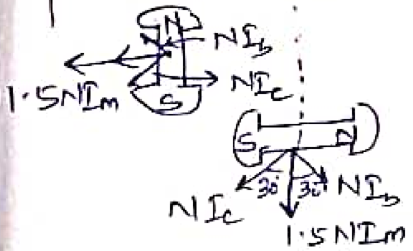
→ It is the effect of armature flux on the main field flux, but in alternators the power factor has a considerable effect on the armature reaction.

(i) when load p.f is unity (ii) when p.f is zero lagging (iii) when p.f is zero leading.









∴ Armature mmf $= NI_m + 2(\frac{1}{2}NI_m)\cos 60° = 1.5NI_m$

At t_1, the mmf of the main field is upwards & the armature mmf is behind it by 90 electrical degree.

Similarly at t_2.

Armature mmf $= 2 \times 0.866 NI_m \times \cos 30° = 1.5NI_m$

NOTE → (i) Armature mmf remains constant with time.
(ii) It is 90 space degree behind the main field mmf.
(iii) It rotates synchronously round the armature.

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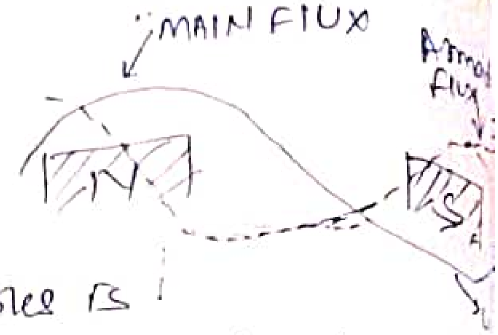
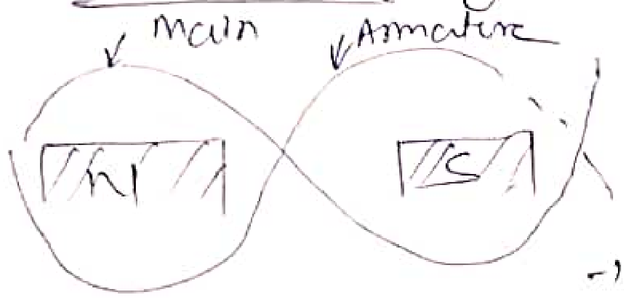
1. UNITY POWER FACTOR



→ The armature flux is cross-magnetising. The result is that the flux at the leading tips of the poles is reduced while it is increased at the trailing tips.

2. Zero p.f. Lagging



The armature flux is in direct opposition to the main flux.

→ The main flux is decreased

→ Armature reaction is wholly demagnetising.

3. Zero p.f. leading.

→ In this case Armature reaction is wholly magnetising. which results in greater induced emf.

Synchronous Reactance

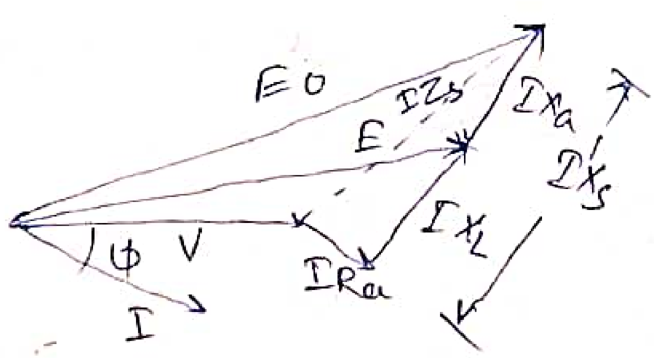
⇒ Terminal voltage is decreased from its no load value E_0 to V, because

(i) drop due to armature resistance IR_a.

(ii) drop due to leakage reactance IX_L.

(iii) drop due to armature reaction.

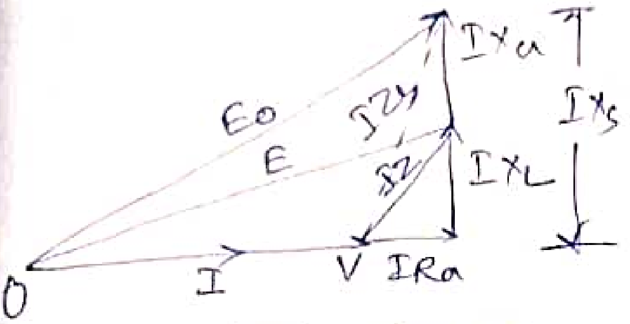
$X_s = X_L + X_a$



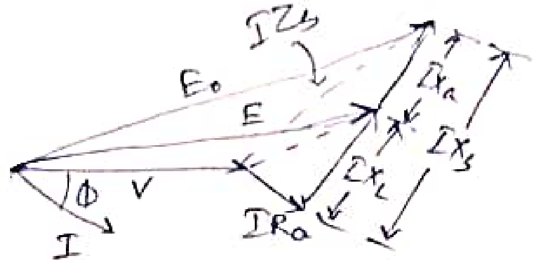
VECTOR DIAGRAM OF A LOADED ALTERNATOR

Ref! $E_0 \to$ No load emf

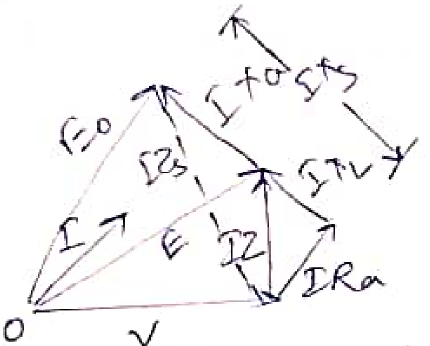
$E \to$ Load induced emf. E is vectorially less than E_0 by IX_a.



(a) unity P.F.



(b) lagging P.F



(c) leading p.f.

$V \to$ Terminal voltage, vectorially less than E_0 by IZ_s.

$Z = \sqrt{R_a^2 + X_L^2}$

$I \to$ armature current/phase, $\phi \to$ load p.f. angle.

Voltage Regulation

→ The voltage regulation of an alternator is defined as the rise in voltage when full-load is removed divided by the rated terminal voltage.

$\therefore$ % regulation up $= \frac{E_0 - V}{V} \times 100$.

DETERMINATION OF VOLTAGE REGULATION

Three methods → (i) Synchronous Impedance or Emf method

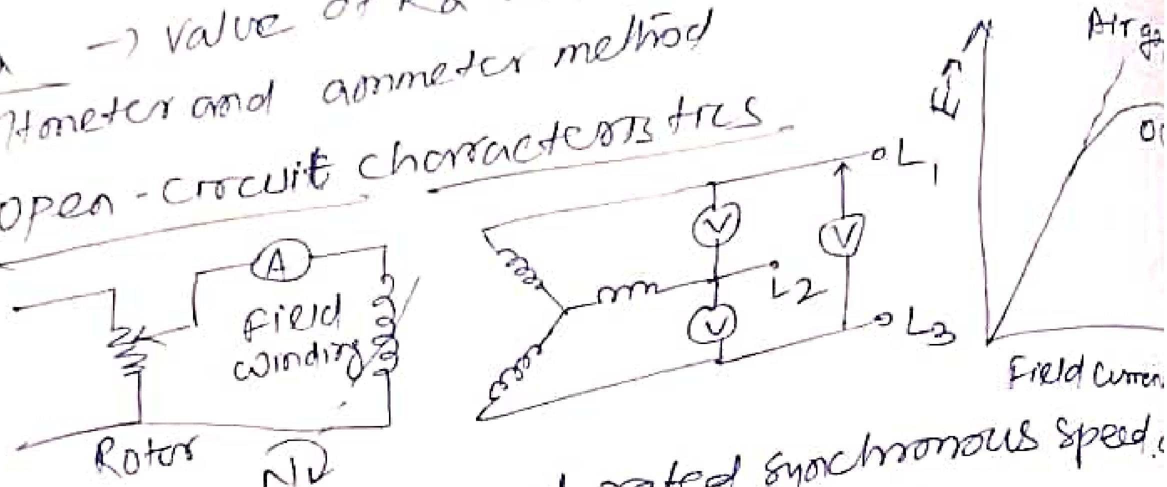
(ii) Ampere-turn or mmf method

(iii) zero power factor or Potier method.

All these methods require
ⓐ Armature resistance (Ra
ⓑ open-circuit/No-load charateristic.
ⓒ short-circuit characteristic.

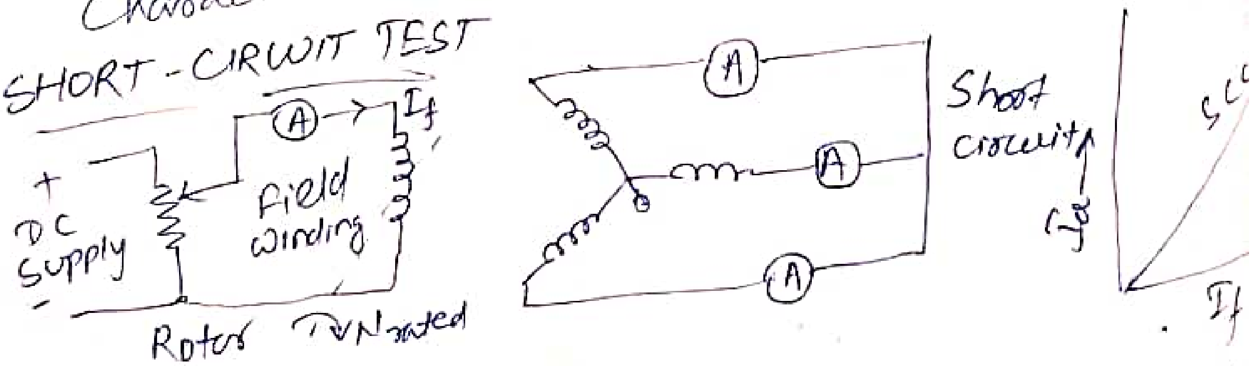
ⓐ Ra → value of Ra can be measured directly by Voltmeter and ammeter method

ⓑ open-circuit characteristics.



The alternator is run at rated synchronous speed and Load terminals are kept open.
→ Then the field is gradually increased and the terminal Voltage E_f is measured at each step.
→ The excitation current may be increased to get 25% more than rated voltage of the alternator.
→ The curve so obtained is called open-circuit characteristic (O.C.C.)

SHORT-CIRCUIT TEST



→ The armature terminals are shorted through ammeters.
→ Field current may be increased to get armature currents upto 150% of the rated value.
→ The characteristics obtained is called SCC

Synchronous Impedance Method

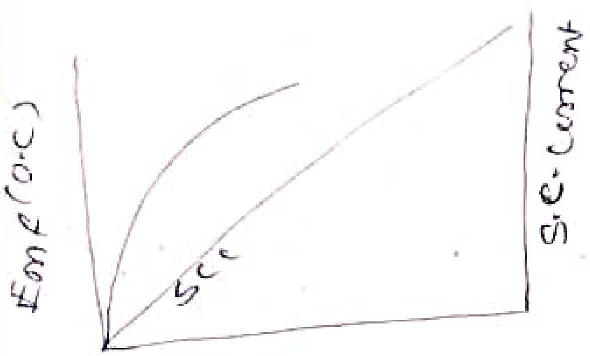
Field current $E_1 = I_1 Z_s$

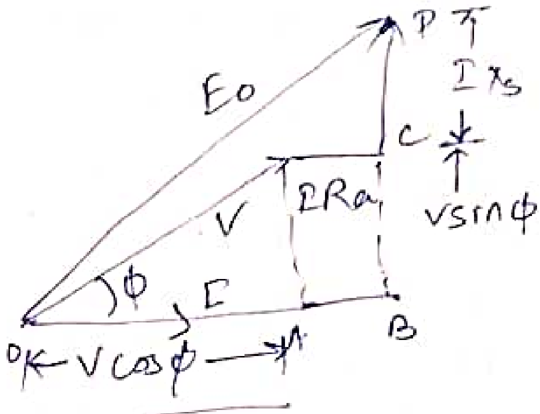
$$\therefore Z_s = \frac{E_1 \text{ (Open-circuit)}}{I_1 \text{ (Short-circuit)}}$$

$E_1 \rightarrow$ open circuit voltage

$I_1 \rightarrow$ short circuit current.

$X_s = \sqrt{Z_s^2 - R_a^2}$





$OD = E_0$ $\therefore E_0 = \sqrt{OB^2 + BD^2}$

$$E_0 = \sqrt{(V\cos\phi + IR_a)^2 + (V\sin\phi + IX_s)^2}$$

% regn up $= \frac{E_0 - V}{V} \times 100$.

PARALLEL OPERATION OF ALTERNATORS

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as Synchronizing.

Condition $\rightarrow$ (i) The terminal voltage of the incoming alternator must be the same as bus bar voltage.

(ii) The speed of the incoming alternator must be such that its frequency equals busbar frequency

(iii) The phase of the alternator voltage must be identical with the phase of bus bar voltage.

(iv) Phase sequence must be same.

ADVANTAGES OF PARALLEL OPERATION

(i) Continuity of service

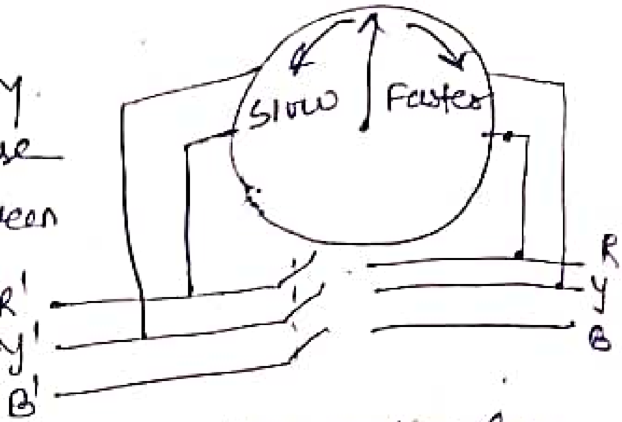
(ii) Efficiency

(iii) Maintenance & Repair

(iv) Load growth.

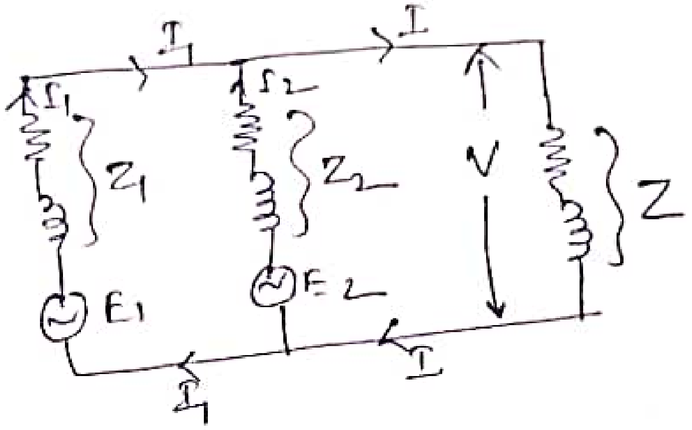
Methods of Synchronisation

Synchroscope method



It is an instrument that indicated by means of a revolving pointer the phase difference & frequency difference between the voltages of the incoming alternator & the busbar. It is a small motor field being supplied from the busbars through a potential transformer & the rotor from the incoming alternator. → A pointer is attached to the rotor → When the ~~incoming~~ incoming alternator is running fast the rotor & hence the pointer moves in clockwise direction. → When the incoming Alternator is running slow the pointer moves in anticlockwise direction. → When the frequency of the incoming alternator is equal to that of the busbars no torque acts on the rotor & the pointer points vertically → It indicates the correct instant for connecting the incoming alternator due to the busbars. → It is superior to Lamp method.

SHARING OF LOAD CURRENTS BY TWO ALTERNATORS IN PARALLEL



let $E_1, E_2 \to$ Induced emfs/phase

$Z_1, Z_2 \to$ Synchronous Impedance/phase

$Z \to$ load impedance/phase

$I_1, I_2 \to$ Currents supplied by two machines.

$V \to$ common terminal voltage/phase

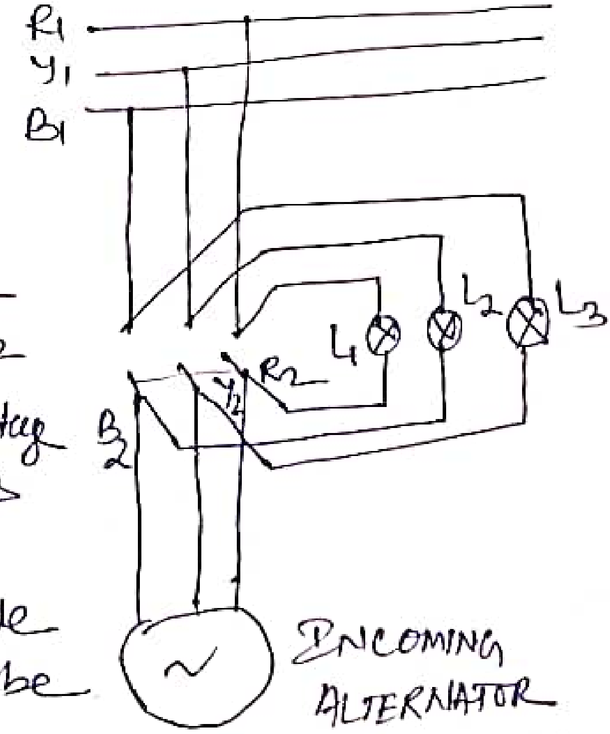
$$V = E_1 - I_1 Z_1 = E_2 - I_2 Z_2$$

$$I_1 = \frac{E_1 - V}{Z_1} \quad \therefore I_2 = \frac{E_2 - V}{Z_2}$$

$$I = I_1 + I_2 = \frac{E_1 - V}{Z_1} + \frac{E_2 - V}{Z_2} \Rightarrow V = (I_1 + I_2)Z = IZ$$

circulatory current on no load $I_c = \frac{E_1 - E_2}{Z_1 + Z_2}$

Three Lamp method or DARK BRIGHT METHOD



Three lamps are used for this method.

- L_1 is straight connected between the corresponding phase (R_1 & R_2)
- L_2 is connected between Y_1 & B_2
- L_3 is connected between B_1 & Y_2
- when frequency & phase of the voltage of the incoming alternator is same as that of the busbars the straight connected lamp L_1 will be dark while cross connected lamps L_2 & L_3 will be equally bright. At this instant the synchronisation is perfect.
- At the instant when R_1 is in phase with R_2, voltage across lamp L_1 is zero & voltages across lamps L_2 & L_3 are equal.

INDUCTION MOTOR

CLASSIFICATION OF A.C. MOTOR

1. AS REGARDS THEIR PRINCIPLE OF OPERATION.
   (a) Synchronous
   (b) Asynchronous → (i) Induction motor.
2. As per TYPE OF current → (i) single phase
   (ii) Three phase
3. As their speed → (i) constant speed
   (ii) variable speed
   (iii) adjustable speed

Induction Motor

Principle → In ac motors the rotor does not receive electric energy by conduction but by induction as the secondary of a 2-winding transformer receives its power from the primary. That is why such motors are known as induction motors and can be treated as a rotating transformer.

Advantages → (i) It has very simple & rugged, almost unbreakable construction.
(ii) Its cost is low and it is very reliable.
(iii) It has high efficiency.
(iv) It requires minimum of maintenance.

Disadvantage → (i) speed cannot be varied. or
(ii) Its speed decreases with increase in load.

Construction

Induction motor consists of two main parts.
(a) stator (b) rotor.

Stator

It is made up of a number of stampings, which [are] slotted to receive the windings. The stator carries a 3-phase winding and is supplied with 3-phase su[pply]

-> Greater the number of poles, lesser the speed.

-> When supplied with 3-phase currents, produce [a] magnetic flux, which is of constant magnitude, but revolves at synchronous speed ($N_s = \frac{120f}{P}$)

-> This revolving magnetic flux induces an e.m.f in the rotor by mutual induction.

Rotor

(i) squirrel cage rotor.
(ii) Phase wound rotor.

Squirrel Cage Rotor

-> Almost 90 percent of induction motors are squirrel type, because this type of rotor ~~of motor~~ has the simplest & most rugged construction.

-> The rotor consists of a cylindrical laminated core with parallel slots for carrying rotor conductors. cores consists of heavy ~~bars~~ bars of copper, aluminium or al[loy]

-> The rotor bars are permanently short-circuited on themselves, hence it is not possible to add any exter[nal] resistances in series.

-> The rotor slots are usually slight skewed ~~[illegible]~~

(i) It helps to make the motor run quietly by reducing the magnetic hum.

(ii) It helps in reducing the locking tendency.

Phase wound rotor

→ This type of rotor is provided with 3-phase double layer distributed winding.

→ The rotor is wound for as many poles as the number of stator poles

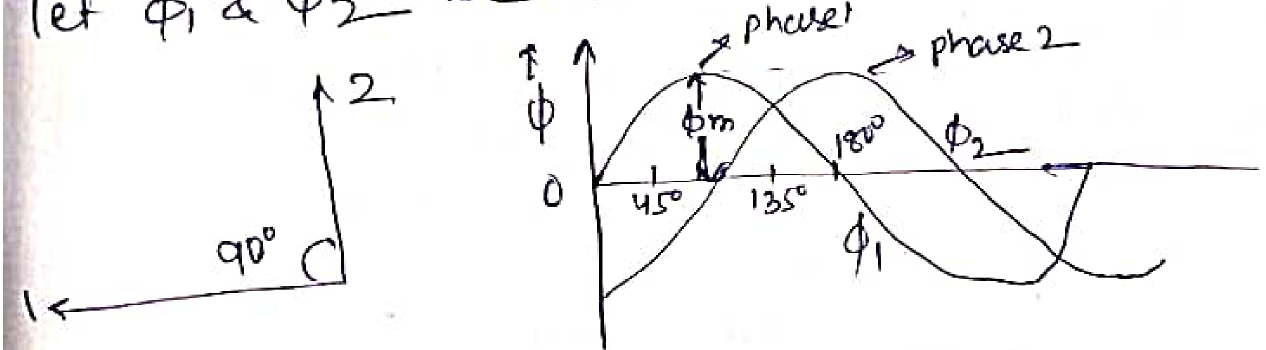
→ Under normal running condition the slip rings are automatically short circuited by means of a metal collar, which is pushed along the shaft and connects all the rings together.

1. Frame
2. Stator and Rotor core.
3. Stator and rotor windings.
4. Air gap
5. Fans
6. slip rings

PRODUCTION OF ROTATING FIELD

TWO phase supply: → let 2-ϕ, 2-pole stator having two identical windings, 90 space degrees apart.

let ϕ_1 & ϕ_2 be the instantaneous values of the fluxes.



(a) when $\theta = 0°$ $\quad \phi_1 = 0$ but $\phi_2 =$ maximum

Hence resultant flux $\phi_r = \phi_m$ (negative)

(b) when $\theta = 45°$ $\quad \phi_1 = \phi_m/\sqrt{2}$ (positive)

$\phi_2 = \phi_m/\sqrt{2}$ (but still negative)

so $\phi_r = \sqrt{(\phi_m/\sqrt{2})^2 + (\frac{\phi_m}{\sqrt{2}})^2} = \phi_m$ resultant shifted 45° clockwise.

(c) when $\theta = 90°$ $\quad$ Hence $\phi_2 = 0$ but $\phi_1 = \phi_m$ (positive)

$\phi_r = \phi_m$ (shifted by an angle 45°)

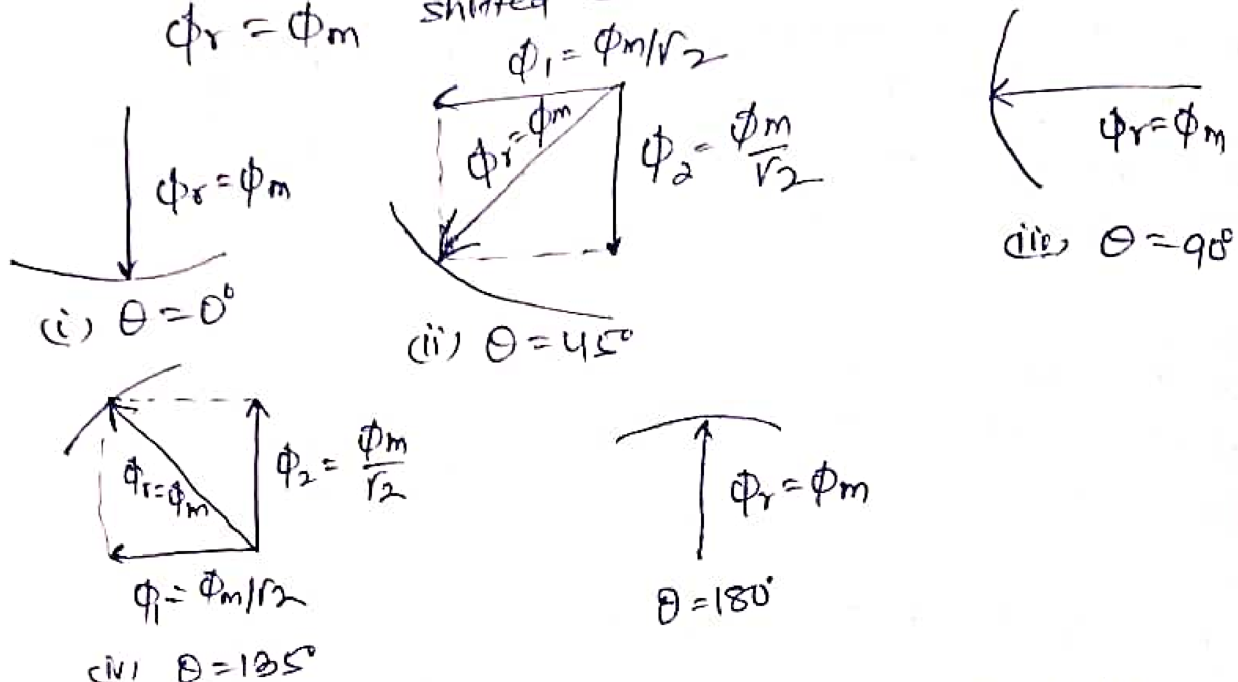
d) when $\theta = 135°$ $\quad \phi_1 = \phi_m/\sqrt{2}$ (positive)

$\phi_2 = \phi_m/\sqrt{2}$ (positive)

$\phi_r = \phi_m$ shifted clockwise by another 45°.

e) when $\theta = 180°$ $\quad \phi_1 = 0$, $\phi_2 = \phi_m$ (positive)

$\phi_r = \phi_m$ shifted clockwise by another 45°.



Conclusion → (i) The magnitude of the resultant flux is constant and is equal to ϕ_m + the maximum flux due to either phase.

(ii) That the resultant flux rotates at synchronous speed $N_s = 120f/p$.

<u>Mathematical Proof</u> let $\phi_1 = \phi_m \sin \omega t$

$$\phi_2 = \phi_m \sin(\omega t - 90°)$$

$$\phi_r^2 = \phi_1^2 + \phi_2^2$$

$$\phi_r^2 = (\phi_m \sin \omega t)^2 + [\phi_m \sin(\omega t - 90°)]^2$$

$$= \phi_m^2 (\sin^2 \omega t + \cos^2 \omega t) = \phi_m^2$$

$$\therefore \phi_r = \phi_m$$

Three Phase Supply

let us supplied with 3-ϕ supply displaced in space by 120°.

let ϕ_m → maximum value of flux.

ϕ_r → resultant flux.

ϕ_1, ϕ_2 & ϕ_3 are three fluxes.

(i) when $\theta = 0°$

Hence $\phi_1 = 0$ $\quad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m \quad \phi_3 = \frac{\sqrt{3}}{2}\phi_m$

$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2}\phi_m = \frac{3}{2}\phi_m$$

(ii) when $\theta = 60°$

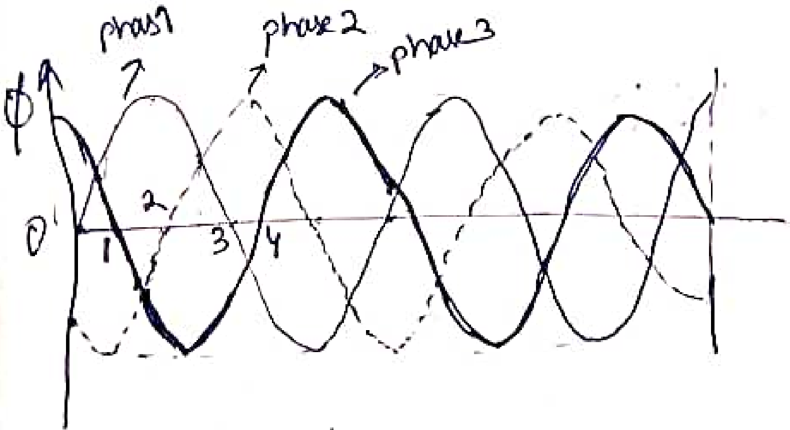
$\phi_1 = \frac{\sqrt{3}}{2}\phi_m \quad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m \quad \phi_3 = 0$

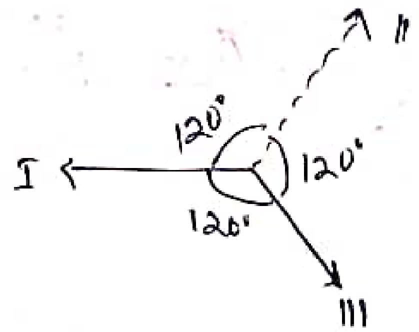
$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos 30° = \frac{3}{2}\phi_m$$

the resultant flux $\frac{3}{2}\phi_m$ but has rotated clockwise through an angle of 60°.

(iii) when $\theta = 120°$

Here $\phi_1 = \frac{\sqrt{3}}{2}\phi_m \quad \phi_2 = 0, \quad \phi_3 = -\frac{\sqrt{3}}{2}\phi_m$





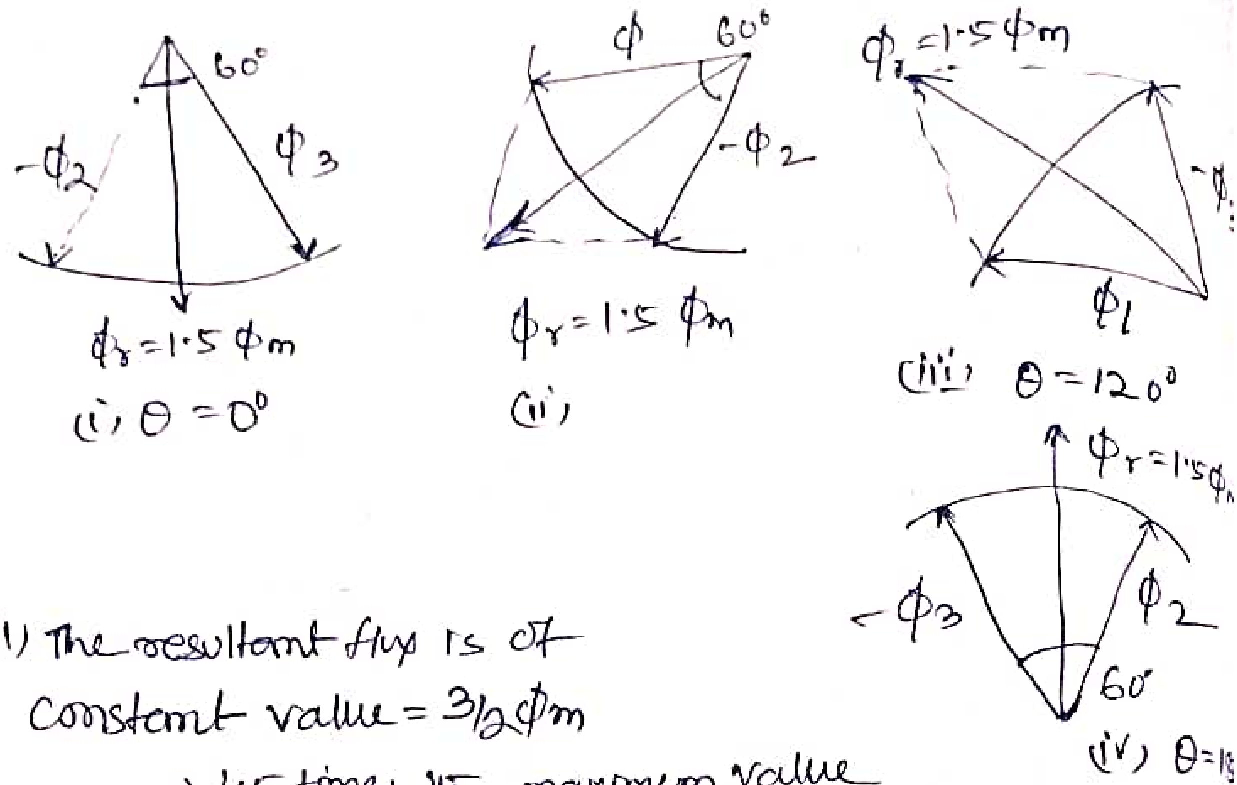
$\phi_r = 3/2\,\phi_m$

The resultant is again same but has further rotated clockwise through an angle of 60°.

(iv) when $\theta = 180°$

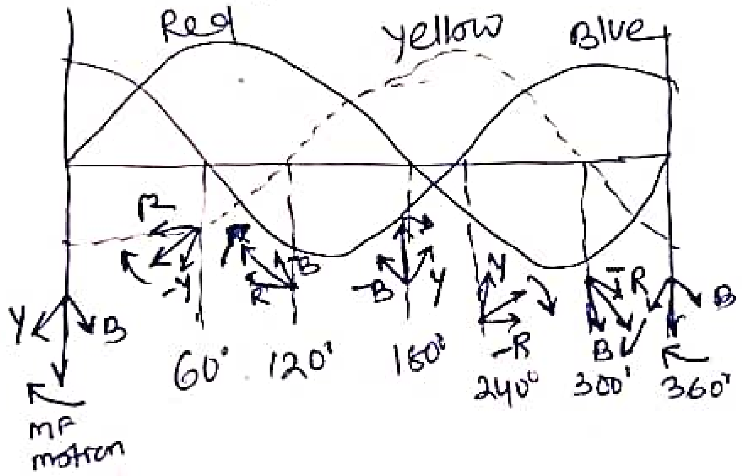
$\phi_1 = 0 \quad \phi_2 = \frac{\sqrt{3}}{2}\phi_m \quad \phi_3 = -\frac{\sqrt{3}}{2}\phi_m$

resultant $3/2\,\phi_m$ has rotated clockwise through an additional angle 60°.



(1) The resultant flux is of constant value $= 3/2\,\phi_m$
→ 1.5 times the maximum value of the flux.

(2) the resultant flux rotates around the stator at synchronous speed $N_s = 120f/p$.



$$\phi_1 = \phi_m (\cos 0° + j\sin 0°) \sin\omega t$$

$$\phi_2 = \phi_m (\cos 240° + j\sin 240) \sin(\omega t - 120°)$$

$$\phi_3 = \phi_m (\cos 120° + j\sin 120°) \sin(\omega t - 240°)$$

$$\phi_r = \frac{3}{2}\phi_m (\sin\omega t + j\cos\omega t) = \frac{3}{2}\phi_m \angle 90° - \omega t$$

Why Does the Rotor Rotate ?

-> When the 3-ϕ stator windings are supplied with 3-ϕ, a magnetic flux of constant magnitude but rotating at synchronous speed is set up.

-> The flux ~~passes~~ passes through the air gap and so cuts the ~~rotor conductors which are~~ stationary rotor conductors.

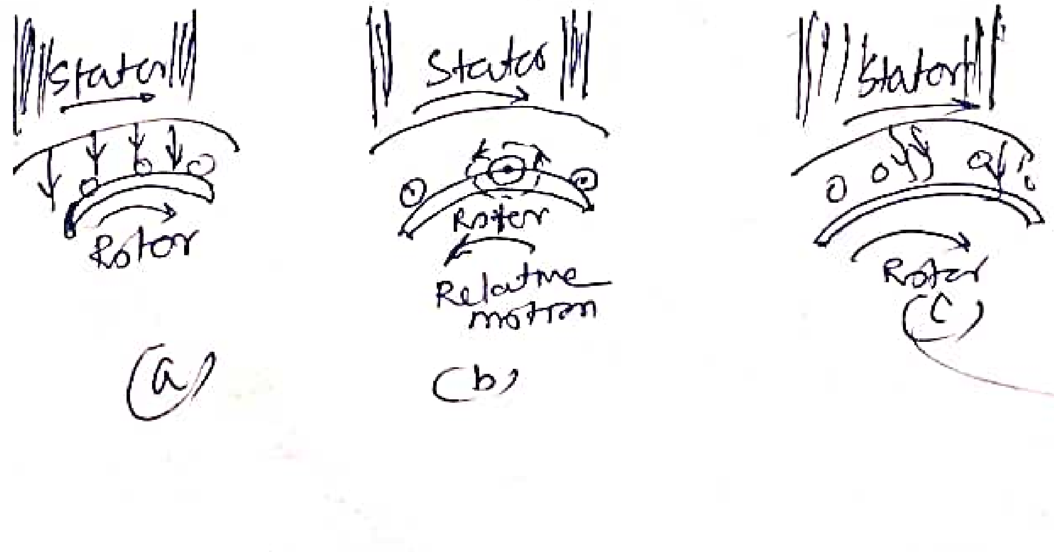
-> Due to the relative speed between the rotating flux and stationary conductors an emf is induced in it.

-> The frequency of the induced emf is the same as the supply frequency.

-> The rotor current is produced whose direction is given by Lenz's law, is such as to oppose the very cause producing it.

-> Hence to reduce the relative speed the rotor starts running in the same direction as that of the flux and tries to ~~catch~~ catch up with the rotating flux.

-> The relative motion of the rotor with respect to the stator is anticlockwise.



Slip → The difference between synchronous speed N_s and the actual speed N of the rotor is known as slip.

$$\% \text{ slip } s = \frac{N_s - N}{N_s} \times 100$$

$N_s - N$ → ~~sp~~ slip speed.

rotor speed $N = N_s(1-s)$

Frequency of rotor current

let f' → frequency of rotor current.

$$N_s - N = \frac{120 f'}{P} \qquad N_s = \frac{120 f}{P}$$

$$\frac{f'}{f} = \frac{N_s - N}{N_s} = s \quad \Rightarrow f' = sf$$

Speed of the rotor field in space = Speed of rotor magnetic field relative to rotor + sp of rotor relative to space.

$$= sN_s + N = sN_s + N_s(1-s) = N_s.$$

Ex A 4pole, 3-φ induction motor operates from supply whose frequency is 50Hz. Calculate
(i) the speed at which the magnetic field of the stator is rotating.
(ii) the speed of the rotor when the slip is 0.04.
(iii) The frequency of rotor current when the slip is 0.(

soln: $N_s = 120f/P = 120 \times 50/4 = 1500$ rpm
(ii) rotor speed $N = N_s(1-s) = 1500(1-0.04) = 14$
(iii) frequency of rotor current $f' = sf = 0.03 \times 50 = 1$
$= 90$
(iv) since at standstill $s = 1$ $f' = sf = 1 \times f = 50$Hz

(28)

Relation Between Torque and Rotor Power factor

$T_a \rightarrow$ Armature torque

$T_a \propto I_a \phi$

$I_a \rightarrow$ Armature current

$\phi \rightarrow$ flux

In Induction motor

$$T \propto \phi I_2 \cos\phi_2$$

$$T = K \phi I_2 \cos\phi_2$$

where I_2 = rotor current at standstill

ϕ_2 = angle between rotor emf and rotor current.

$K \rightarrow$ a constant.

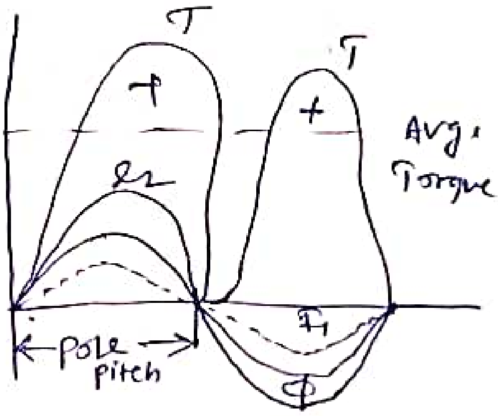
~~...~~ let $E_2 \rightarrow$ rotor emf at standstill

$$E_2 \propto \phi$$

$$\therefore T \propto E_2 I_2 \cos\phi_2$$

$$\text{or } T = K_1 E_2 I_2 \cos\phi_2$$

$K_1 \rightarrow$ another constant.



→ It is cleared that as ϕ_2 increases ($\cos\phi_2$ decreases) the torque decreases and vice versa.

→ The revolving flux ~~distributions~~ induces in each rotor conductor or bar an emf whose value depends on the flux density. Hence the induced emf ($e = Blv$ volt) in the rotor is also sinusoidal.

(i) Rotor assumed Non-inductive ($\phi_2 = 0$)

→ The rotor current

Starting Torque

let E_2 → rotor emf per phase at stand still
R_2 → rotor resistance/phase
X_2 → rotor reactance/phase at stand still
X_2 → rotor impedance/phase at stand still.

$Z_2 = \sqrt{R_2^2 + X_2^2}$ = rotor impedance/phase

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2+X_2^2)}}$$

$$\cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2+X_2^2}}$$

Starting torque $T_{st} = K_1 E_2 I_2 \cos\phi_2$

$$\boxed{T_{st} = K_1 E_2 \frac{E_2}{\sqrt{R_2^2+X_2^2}} \times \frac{R_2}{\sqrt{R_2^2+X_2^2}} = \frac{K_1 E_2^2 R_2}{R_2^2+X_2^2}}$$

If supply voltage V is constant, then the flux & and hence E_2 both are constant.

$$\boxed{T_{st} = K_2 \frac{R_2}{R_2^2+X_2^2} = K_2 \frac{R_2}{Z_2}}$$

where K_2 → another constant.

$K_1 = \frac{3}{2\pi N_s}$ ∴ $\boxed{T_{st} = \frac{3}{2\pi N_s} \frac{E_2^2 R_2}{R_2^2+X_2^2}}$

Starting torque of a Squirrel cage Motor

→ In squirrel cage motor resistance is fixed and small as compared to its reactance so frequency of the rotor currents equals the supply frequency at stand still. so starting current I_2 of rotor is very large in magnitude, lags by a very large angle behind E_2.

- So starting torque per amp is very poor.
- It is almost 1.5 times the full load torque although the starting current is 5 to 7 times the full load current.

Starting Torque of a Slip ring Motor

Condition for Maximum starting Torque

$$T_{st} = \frac{3}{2\pi N_s} \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

$$T_{st} = \frac{K_2 R_2}{R_2^2 + X_2^2}$$

$$\frac{dT_{st}}{dR_2} = K_2 \left[\frac{1}{R_2^2 + X_2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$\therefore R_2 = X_2$

$$R_2^2 + X_2^2 = 2R_2^2 \Rightarrow \boxed{R_2 = X_2}$$

EFFECT OF CHANGE IN SUPPLY VOLTAGE ON STARTING TORQUE

$$T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

$E_2 \propto$ supply voltage V.

$$T_{st} = \frac{K_3 V^2 R_2}{R_2^2 + X_2^2} = \frac{K_3 V^2 R_2}{Z_2^2}$$

$T_{st} \propto V^2$.

Rotor Emf and Reactance under Running Condition

let $E_2 \rightarrow$ standstill rotor induced emf/phase

$X_2 \rightarrow$ " rotor reactance/phase.

$f_2 \rightarrow$ rotor current frequency at standstill when rotor starts rotating the relative speed between it and the rotating stator flux is decreased hence the rotor induced emf is directly proportional to this relative speed is also decreased.

Under running condition $\boxed{E_r = SE_2}$

frequency $\boxed{f_r = Sf_2}$

$\boxed{X_r = SX_2}$

TORQUE UNDER RUNNING CONDITION

$T \propto E_r I_r \cos\phi_2$

or $T \propto \phi I_r \cos\phi_2$

$E_r \rightarrow$ rotor emf/phase under running condition

$I_r \rightarrow$ rotor current/phase " " ".

$E_r = SE_2$

$$I_r = \frac{E_r}{Z_r} = \frac{SE_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2 + (SX_2)^2}}$$

$$T \propto \frac{S\phi E_2 R_2}{R_2^2 + (SX_2)^2} = \frac{K\phi S E_2 R_2}{R_2^2 + (SX_2)^2}$$

$$T = \frac{K_1 S E_2^2 R_2}{R_2^2 + (SX_2)^2} \quad (\because E_2 \propto \phi)$$

$$T = \frac{3}{2\pi N_s} \cdot \frac{SE_2^2 R_2}{R_2^2 + (SX_2)^2}$$

$$= \frac{3}{2\pi N_s} \frac{SE_2^2 R_2}{Z_r^2}$$

when $s = 1$

$$\boxed{T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + X_2^2}} \quad \boxed{\text{or} = \frac{3}{2\pi N_s} \frac{E_2^2 R_2}{R_2^2 + X_2^2}}$$

Example

The star connected rotor of an induction motor. has a standstill impedance of $(0.4+j4)$ ohm/phase. & rheostat impedance per phase $(6+j2)$ ohm. The motor has an induced emf of 80V between slip rings at stand still when connected to its normal supply voltage.

(i) rotor current at standstill with the rheostat is in the circuit

(ii) when the slip rings are short circuited & motor is running with a slip of 3%.

Soln: ① stand still condition

voltage/rotor phase $= 80/\sqrt{3} = 46.2$ V.

Stator impedance/phase $= (6.4+j6)$
$= 8.77 \angle 43.15°$

Rotor current/phase $= 46.2/8.77 = 5.27$ A.

$pf = \cos 43.15 = 0.729$

② Running Cond^n

Rotor voltage/phase $= E_r = SE_2 = 0.03 \times 46.2 = 1.386$ V

Reactance/phase $X_r = SX_2 = 0.03 \times 4 = 0.12$

$Z_r = 0.4 + j0.12 = 0.4176 \angle 16.7°$

Rotor current/phase $= 1.386/0.4176 = 3.32$ A

$pf = \cos 16.7 = 0.96$

Condition for maximum torque under running

$$T = \frac{K\phi S E_2 R_2}{R_2^2 + (SX_2)^2} = K_1 \frac{S E_2^2 R_2}{R_2^2 + (SX_2)^2}$$

for maxm torque differentiating the above expression w.r.t S, let $y = 1/T$

$$y = \frac{R_2^2 + (SX_2)^2}{K\phi S E_2 R_2} = \frac{R_2}{K\phi S E_2} + \frac{S X_2^2}{K\phi E_2 R_2}$$

$$\frac{dy}{ds} = \frac{-R_2}{K\phi S^2 E_2} + \frac{X_2^2}{K\phi E_2 R_2} = 0$$

$$\Rightarrow \frac{R_2}{K\phi S^2 E_2} = \frac{X_2^2}{K\phi E_2 R_2} \quad \text{or } R_2^2 = S^2 X_2^2 \text{ or } \boxed{R_2 = SX_2}$$

slip corresponding to maximum Torque $S = R_2/X_2$

putting $R_2 = SX_2$

$$T_{max} = \frac{K\phi S^2 E_2 X_2}{2S^2 X_2^2} \text{ or } \frac{K\phi S E_2 R_2}{2R_2^2} \text{ or } T_{max} = \frac{K\phi E_2}{2X_2}$$

substituting $S = R_2/X_2$

$$T = K_1 \frac{S E_2^2 R_2}{R_2^2 + (SX_2)^2} \text{ or } T_{max} = \frac{K_1 (R_2/X_2) E_2^2 R_2}{R_2^2 + \left(\frac{R_2}{X_2}\right)^2 X_2^2}$$

$$T_{max} = K_1 \frac{E_2^2}{2X_2}$$

$$\boxed{T_{max} = \frac{3}{2\pi N_s} \frac{E_2^2}{2X_2} \ \text{N-m}}$$

i) The maximum torque is independent of rotor resistance.

ii) Maximum torque varies inversely as standstill reactance.

, maximum torque varies directly as the square of the applied voltage.

ROTOR TORQUE AND BREAKDOWN TORQUE

$$T = T_b \left[\frac{2}{(S_b/S) + (S/S_b)} \right]$$

$T_b \rightarrow$ Breakdown torque

$S_b \rightarrow$ breakdown or pull out slip.

Ex → Calculate the torque exerted by an 8-pole 50 Hz, 3 phase induction motor operating with a 4% slip which develops a maximum torque of 150 kg-m at a speed of 660 rpm. Resistance per phase of the rotor is 0.5 Ω.

Soln.

$$N_s = 120 \times \frac{50}{8} = 750 \text{ rpm}$$

Speed at maximum torque = 660 rpm

$$S_b = \frac{750 - 660}{750} = 0.12$$

For maximum torque $R_2 = S_b X_2$

$$X_2 = R_2/S_b = 0.5/0.12 = 4.167\,\Omega$$

$$T_{max} = K\phi E_2 \frac{S_b}{2R_2} = K\phi E_2 \frac{0.12}{2 \times 0.5} = 0.12 K\phi E_2$$

When slip is 4%

$$T = K\phi E_2 \frac{SR_2}{R_2^2 + (SX_2)^2} = K\phi E_2 \frac{SR_2}{R_2^2 + (SX_2)^2}$$

$$= K\phi E_2 \frac{0.04 \times 0.5}{0.5^2 + (0.04 \times 4.167)^2} = \frac{0.02 K\phi E_2}{0.2778}$$

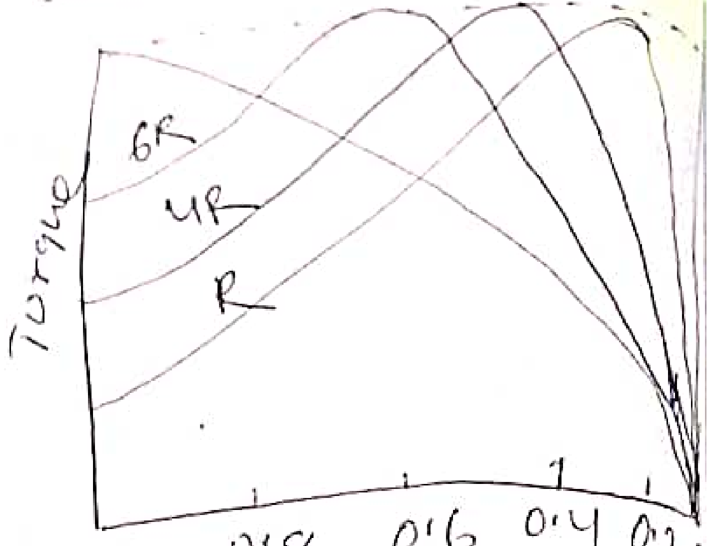
$$\frac{T}{T_{max}} = \frac{T}{150} = \frac{0.02}{0.2778 \times 0.12} \therefore T = 90 \text{ kg-m}$$

"

Relation Between Torque and Slip

$$T = \frac{K\phi S E_2 R_2}{R_2^2 + SX_2^2}$$

When $S=0, T=0$
hence the curve starts from point O.



$T \propto S/R_2$

$\therefore T \propto S$ if $R_2 \rightarrow$ constant

→ For low values of slip the torque/slip curve is approximately a straight line. As slip increases the torque also increases and becomes maximum when $S = R_2/X_2$.

$$T \propto \frac{S}{(SX_2)^2} \propto \frac{1}{S}$$

$\therefore$ Hence torque slip curve is a rectangular hyperbola.

EFFECT OF CHANGE IN SUPPLY VOLTAGE ON TORQUE AND SPEED

$$T = \frac{K\phi S E_2 R_2}{R_2^2 + (SX_2)^2}$$

$E_2 \propto \phi \propto V$ where $V \rightarrow$ supply voltage

Torque at any speed is proportional to the square of the applied voltage.

let V change to V', S to S' and T to T'

Then $\boxed{T/T' = \frac{SV^2}{S'V'^2}}$

Full Load Torque & Maximum Torque

$S_f \rightarrow$ slip corresponding to full load.

$$T_f = \frac{S_f R_2}{R_2^2 + (S_f X_2)^2} \quad \& \quad T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_f}{T_{max}} = \frac{2S_f R_2 X_2}{R_2^2 + (S X_2)^2}$$

dividing bothee numerator & the denominator by X_2^2

$$\frac{T_f}{T_{max}} = \frac{2S_f R_2/X_2}{(R_2/X_2)^2 + S_f^2} = \frac{2aS}{a^2 + S_f^2}$$

where $a = R_2/X_2 = \frac{\text{resistence}}{\text{stand still reactance}}$

Starting Torque & Maximum Torque

$$T_{st} = \frac{R_2}{R_2^2 + X_2^2}$$

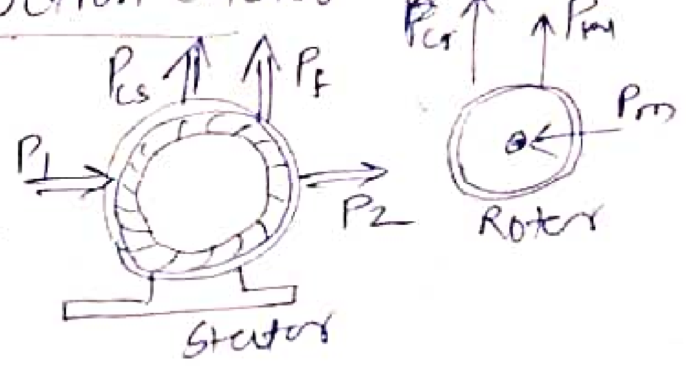
$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{R_2^2 + X_2^2} = \frac{2R_2/X_2}{1 + \left(\frac{R_2}{X_2}\right)^2} = \frac{2a}{1+a^2}$$

$$a = \frac{R_2}{X_2}$$

Plugging of an Induction Motor

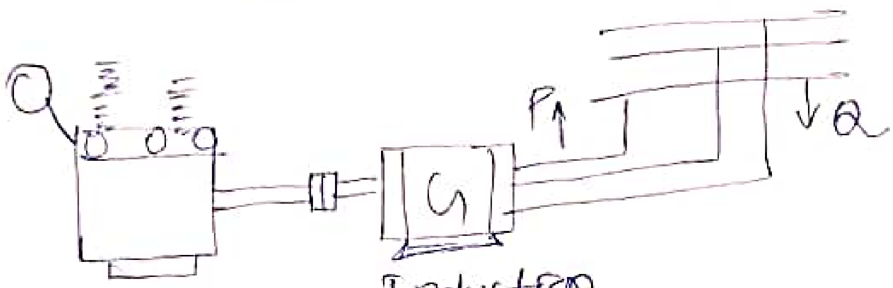
→ An Induction motor can be quickly stopped by interchanging any of its stator leads.



→ It reverses the direction of the revolving flux which produces a torque in reverse direction applying brake on the motor. During plugging period the motor acts as a brake.

Induction motor operating as a Generator

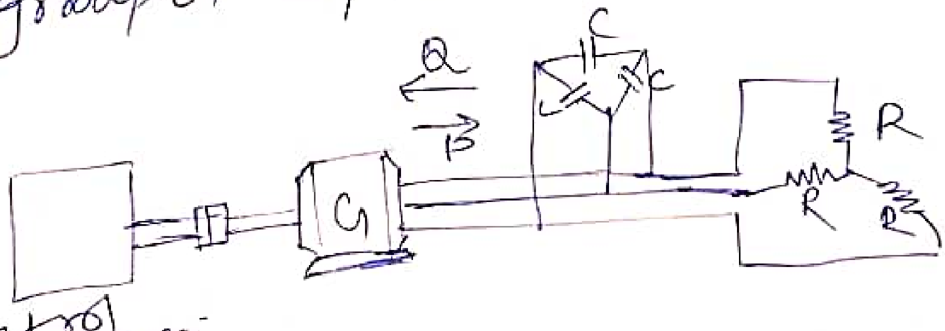
When run faster than its synchronous speed, an induction motor run as a generator called induction generator.



Petrol engine

Induction machine.

The active power is directly proportional to the slip above synchronous speed. The reactive power required by the machine can also be supplied by a group of capacitors.



Petrol engine

→ The terminal voltage increases with capaci

→ Capacitor bank must be large enough to supply the reactive power.

Power Stage in an IM

Motor Input in stator P_1 ⇒ Stator CU & Iron loss ⇒ Rotor input P_2 ⇒ Rotor CU loss ⇒ Mech. Power developed in P_m Gross Torque (T_g) ⇒ windage & friction loss ⇒ Rotor output P_{out}

Torque Developed By an Induction Motor

Motor input in stator P_1

- Stator CU and Iron losses (P_{cs} & P_i)
- rotor input P_2 (via air gap)
  - rotor CU loss (P_{cr})
  - Mechanical Power developed P_m or Gross Torque T_g
    - windage & friction loss (P_w)
    - rotor output (P_{out})

$$T_g = \frac{P_2}{\omega_s} = \frac{P_2}{2\pi N_s}$$ in terms of rotor input

$$T_g = \frac{P_m}{\omega} = P_m/2\pi N$$ in terms of rotor output

T_{sh} → shaft torque is due to power P_{out} which is less than P_m.

$$T_{sh} = P_{out}/\omega = P_{out}/2\pi N$$

T_g and T_{sh} equals the torque lost due to friction and windage loss in motor.

let N and N_s are in rps.

$$T_g = \frac{P_2}{2\pi N_s/60} = \frac{60}{2\pi}\frac{P_2}{N_s} = 9.55\frac{P_2}{N_s} \text{ N·m}$$

$$= \frac{P_m}{2\pi N/60} = \frac{60}{2\pi}\frac{P_m}{N} = 9.55\frac{P_m}{N} \text{ N·m}$$

$$T_{sh} = \frac{P_{out}}{2\pi N/60} = \frac{60}{2\pi}\frac{P_{out}}{N} = 9.55\frac{P_{out}}{N} \text{ N·m}$$

Torque, Mechanical Power and Rotor Output

Stator input P_1 = stator output + stator losses

P_2 = stator output

P_m = rotor input P_2 − rotor cu losses.

Let N rps be the actual speed of the rotor and

& T_g is → N·m

$T_g \times 2\pi N$ = rotor gross output in watts P_m

$$\therefore T_g = \frac{\text{rotor gross output in watts } P_m}{2\pi N} \text{ N-m} \quad \text{—— (1)}$$

If there were no cu loss in rotor.

$$T_g = \frac{\text{rotor input } P_2}{2\pi N_s} \quad \text{—— (2)}$$

Rotor gross output $P_m = T_g \omega = T_g \times 2\pi N$

$P_2 = T_g \omega_s = T_g \times 2\pi N_s$ —— (3)

The difference of two equals rotor cu loss —— (4)

$\therefore$ rotor cu loss = $P_2 - P_m = T_g \times 2\pi(N_s - N)$

from (3) and (4)

$$\boxed{\frac{\text{rotor cu loss}}{\text{rotor input}} = \frac{N_s - N}{N_s} = S}$$

$\therefore$ rotor cu loss = S × rotor input = SP_2

$$\boxed{\text{rotor input} = \frac{\text{rotor cu loss}}{S}}$$

Rotor gross output $P_m = P_2$ − rotor cu loss

= input − S × rotor input

= $(1-S)$ input P_2

$$\boxed{\text{rotor gross output } P_m = (1-S) \text{ rotor input } P_2}$$

$$\frac{\text{rotor gross output } P_m}{\text{rotor input } P_2} = 1-s = \frac{N}{N_s}$$

$$\boxed{\frac{P_m}{P_2} = N/N_s}$$

$$\boxed{\text{rotor efficiency} = \frac{N}{N_s}}$$

$$\boxed{\frac{\text{rotor cu loss}}{\text{rotor gross output}} = \frac{s}{1-s}}$$

Conclusion

$\therefore \quad P_2 : P_m : I^2R :: (1-s) : s$

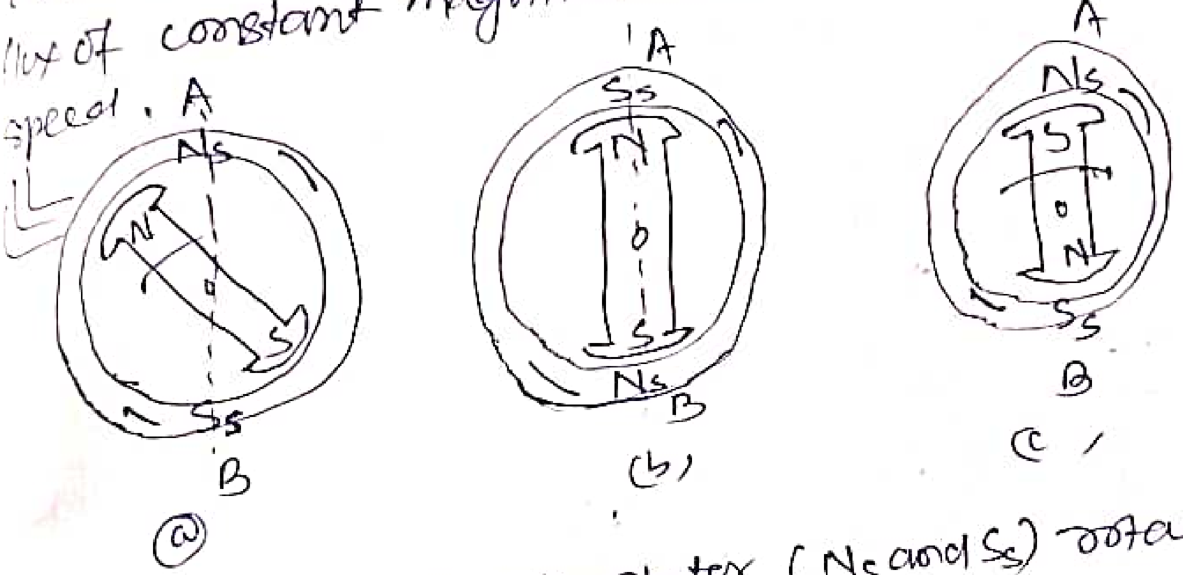
or $P_2 : P_m : P_{cu} :: 1 : (1-s) : s$

SYNCRONOUS MOTOR

CHARACTERISTICS → (i) It runs either at synchronous speed / maintains constant speed. $N_s = 120f/p$.
2. It is not self starting.
3. It is capable of being operated under a wide range of power factor.

PRINCIPLE OF OPERATION

When 3-ϕ supply is given to 3-ϕ winding a magnetic flux of constant magnitude but rotating at synchronous speed.

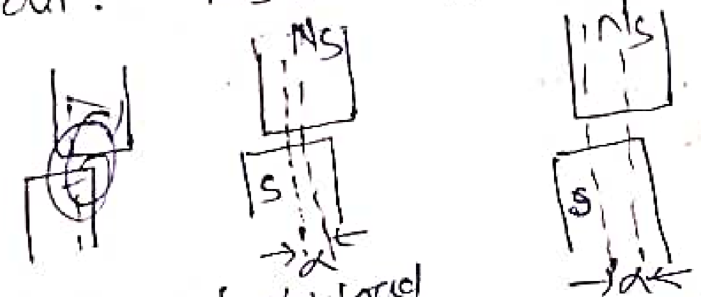


Consider a two pole stator (N_s and S_s) rotating at synchronous speed (clockwise) and are in position A&B.
→ Two similar poles N (rotor) and N_s (stator) as well as S and S_s will repel each other as a result the rotor tends to rotate in anticlockwise.
→ But half a period stator poles having rotated around interchange their positions. N_s is at B and S_s is at A. Under these condition N_s attract S and S_s attract N. Hence rotor tends to rotate clockwise, so due to inertia the rotor cannot respond to such quickly reversing torque and it remains stationary.

→ If the stator and rotor poles are attracting each other, ~~lett~~ is & it rotating clockwise with such a speed that it turns through one pole pitch by the time the stator poles interchange their positions. In fig (b) the stator and rotor poles attract each other. As a result the rotor experience a unidirectional torque in clockwise.

METHOD OF STARTING

→ The rotor is speed up to synchronous speed by some arrangement & then ~~exited~~ excited by the dc source. Because of this interlocking of stator and rotor poles that the motor has either to run synchronously or not at all. $N_s = 120f/p$

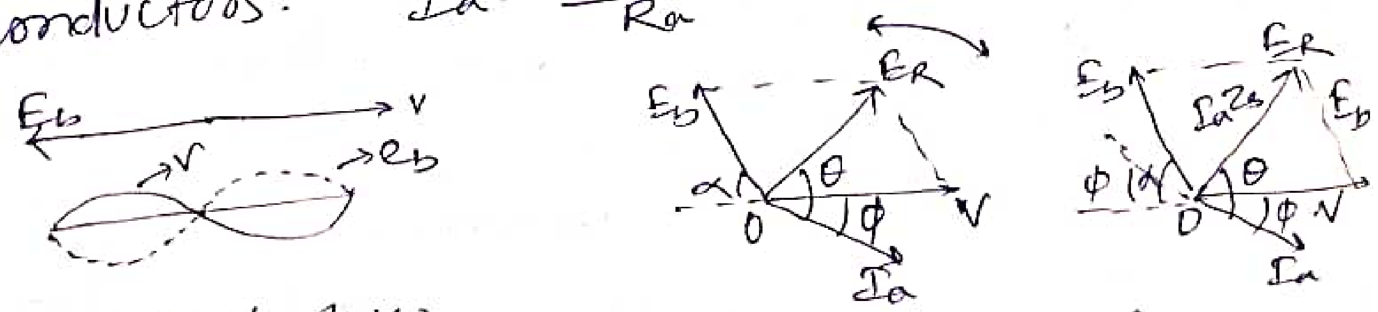


(light load) (α small)

α large Heavy load.

MOTOR ON LOAD WITH CONSTANT ~~CONNECTION~~ EXCITATION

As dc motor E_b → back emf is set up in its armature conductors. $I_a = \frac{V - E_b}{R_a}$



(No load no loss) (No load but loss) (load & losses)

The back emf depends on rotor excitation only. The net voltage in armature (stator) is the vector difference of V & E_b

If motor is on no load but it has losses then the rotor E_b falls back by a certain small angle α so resultant voltage E_R hence current I_a develop.
If the motor is loaded, its rotor will further fall back in phase by a greater value of angle α - called load angle. As a result E_R is increased and motor draws an increased armature current.

POWER FLOW WITHIN A SYNCHRONOUS MOTOR

$R_a \rightarrow$ armature resistance/phase

$X_s \rightarrow$ synchronous reactance/phase.

$Z_s = R_a + jX_s$ $\qquad I_a = \frac{E_R}{Z_s} = \frac{V - E_b}{Z_s}$ $\qquad V = E_b + I_a Z_s$

$\theta \rightarrow$ Internal angle by which I_a lags behind E_R

$\tan\theta = X_s/R_a$

$R_a \rightarrow$ negligible $\theta = 90^\circ$

Motor Input $= V I_a \cos\phi$

3-phase machine $P = \sqrt{3} V_L I_L \cos\phi$

P_m = Backemf × armaturecurrent × cosine of the angle between I_a & E_b

$= E_b I_a \cos(\alpha - \phi)$

Input power $P = P_m + I_a^2 R_a$

$P_m = P - I_a^2 R_a$

$$\boxed{P_m = \sqrt{3} V_L I_L \cos\phi - 3 I_a^2 R_a}$$

Power Input ($P = V I_a \cos\phi$)

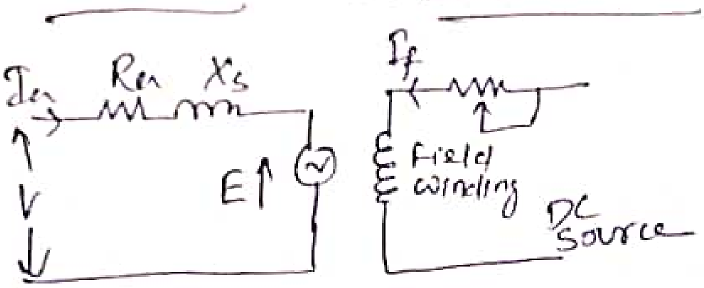
Armature Cu loss $= I_a^2 R_a$

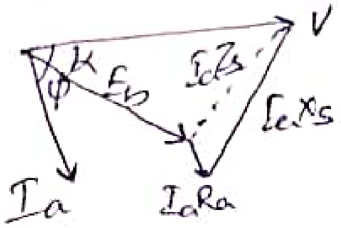
Mechanical power in Armature $P_m = E_b I_a \cos(\alpha - \phi)$

Iron, excitation & Friction losses

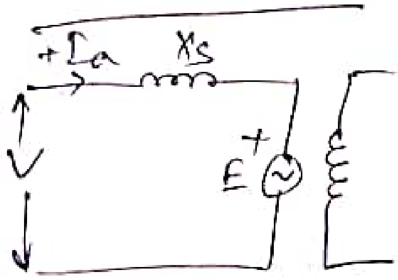
output power P_{out}

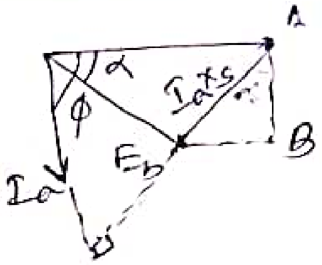
EQUIVALENT CIRCUIT OF A SYNCHRONOUS MOTOR





POWER DEVELOPED BY A SYNCHRONOUS MOTOR





$$AB = E_b \sin\alpha = I_a X_s \cos\phi$$

$$V I_a \cos\phi = \frac{E_b V}{X_s} \sin\alpha .$$

$V I_a \cos\phi$ = motor power input/phase

$$P_{in} = \frac{E_b V}{X_s} \sin\alpha$$

$$= \frac{3 E_b V}{X_s} \sin\alpha .$$

Since stator cu losses have been neglected P_{in} also represents the gross mechanical power (P_m)

$$P_m = \frac{3 E_b V}{X_s} \sin\alpha$$

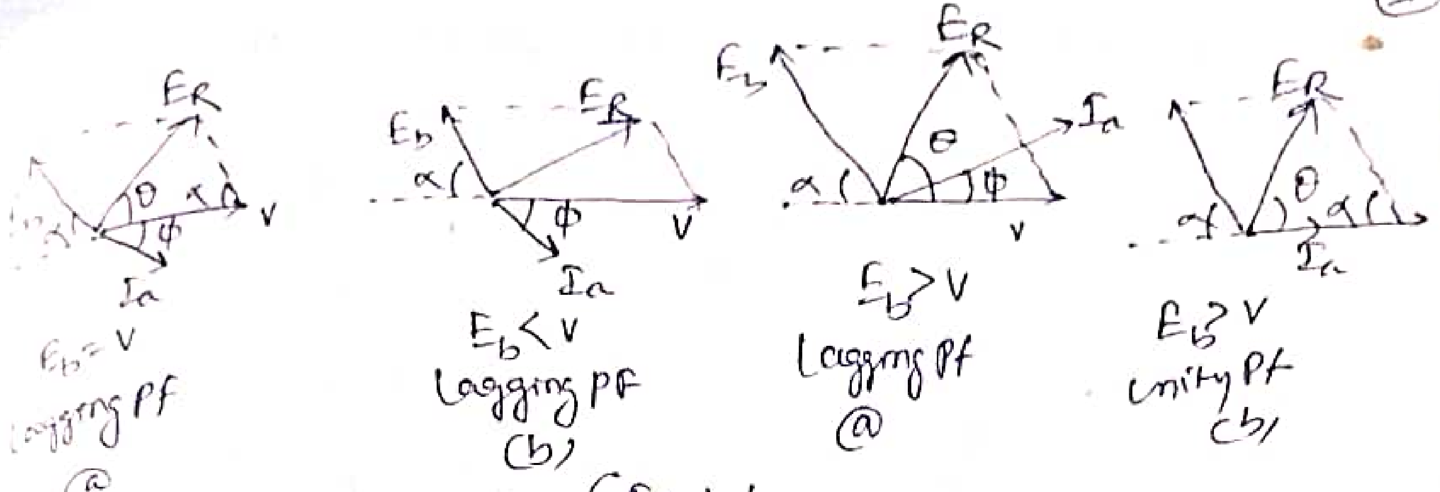
$$T_g = 9.55\, P_m / N_s \ \text{N-m} .$$

SYNCHRONOUS MOTOR WITH DIFFERENT EXCITATIONS

when $E_b = V$ → normal excitation

$E_b < V$ → under excitation

$E_b > V$ → over excitation.



(Fig 1)

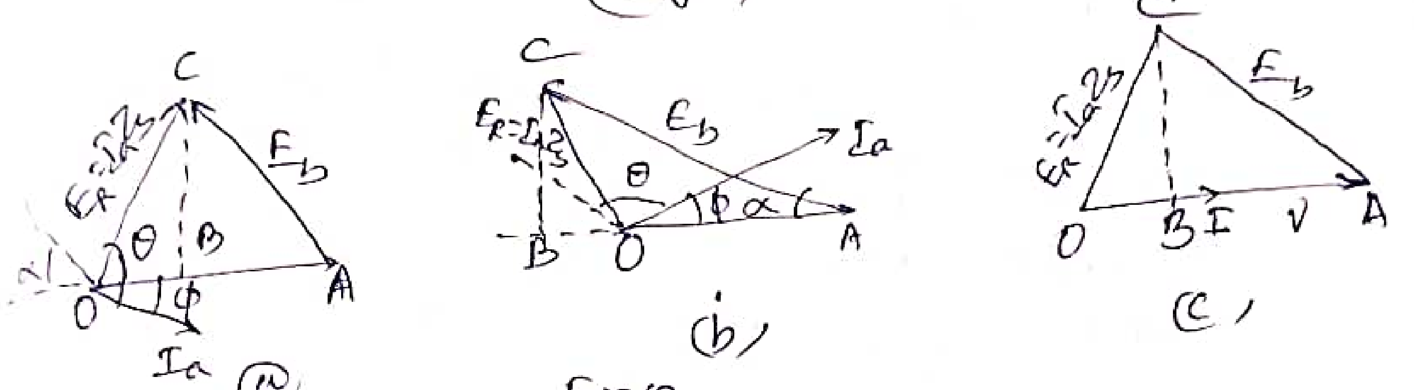


Fig 2

i) Lagging PF from (a) 2

$$AC^2 = AB^2 + BC^2$$

$$= \left[V - E_R \cos(\theta - \phi)\right]^2 + \left[E_R \sin(\theta - \phi)\right]^2$$

$$\Rightarrow E_b = \sqrt{\left[V - I_a Z_s \cos(\theta - \phi)\right]^2 + \left[I_a Z_s \sin(\theta - \phi)\right]^2}$$

ii) ~~Leading~~ P load angle $\alpha = \tan^{-1}\frac{BC}{AB} = \tan^{-1}\left[\frac{I_a Z_s \sin(\theta - \phi)}{V - I_a Z_s \cos(\theta - \phi)}\right]$

ii) Leading PF

$$E_b = V + I_a Z_s \cos[180^\circ - (\theta + \phi)] + j I_a Z_s \sin[180^\circ - (\theta + \phi)]$$

$$\alpha = \tan^{-1} \frac{I_a Z_s \sin[180^\circ - (\theta + \phi)]}{V + I_a Z_s \cos[180^\circ - (\theta + \phi)]}$$

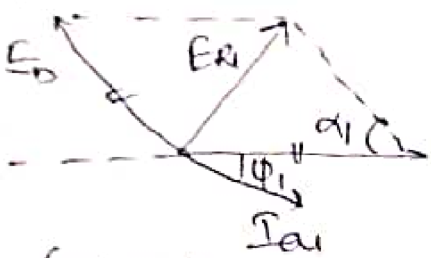
iii) Unity P.F

$OB = I_a R_a$ & $BC = I_a X_s$ $\quad \alpha = \tan^{-1}\left(\frac{I_a X_s}{V - I_a R_a}\right)$

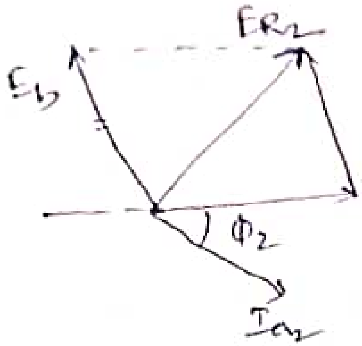
$\therefore E_b = (V - I_a R_a) + j I_a X_s$

EFFECT OF INCREASED LOAD WITH CONSTANT EXCITATION

(i) Normal Excitation



$(E_b = V)$ (a)



(b)

→ When motor is running with light load

(i) torque angle α_1 is small

(ii) E_{R1} is small

(iii) Hence I_{a1} is small (iv) ϕ_1 is small so that $\cos\phi_1$ is large.

If Load on motor is increased

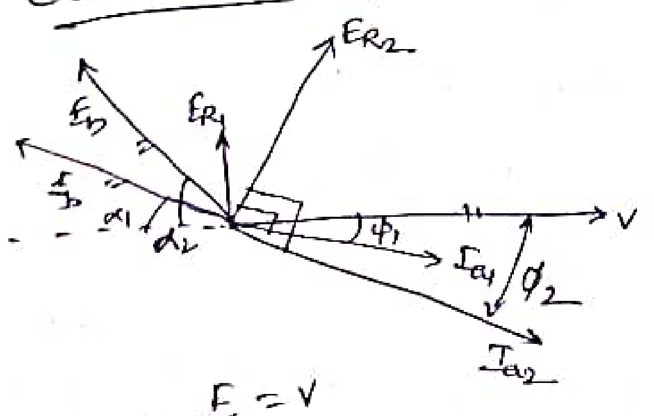
→ (i) rotor falls back in phase, load angle increases to α_2. (Fig b)

→ (ii) The resultant voltage in armature is increased to new value E_{R2}

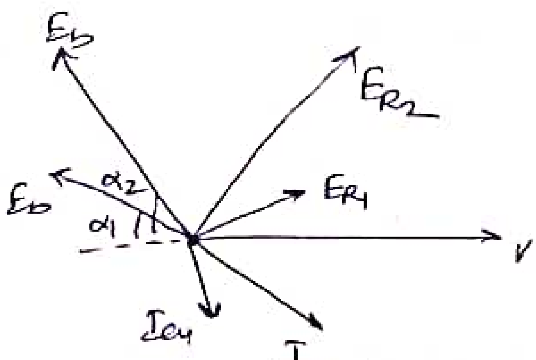
→ (iii) I_{a1} increases to I_{a2} increasing the torque developed by the motor.

→ (iv) ϕ_1 increases to ϕ_2. so that power factor decreases from $\cos\phi_1$ to $\cos\phi_2$.

(ii) Under Excitation



(a) $E_b = V$



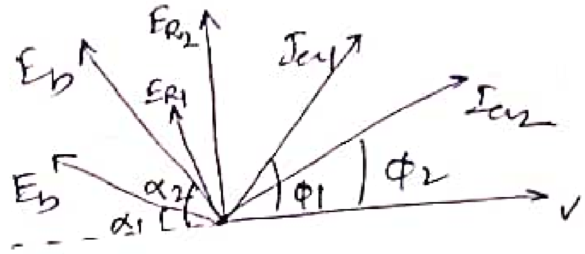
$E_b < V$

(Under excitation)

(b)

As load increases E_{R_1} increases to E_{R_2}, consequently I_{a} increases to I_{a_2} and P.f angle decreases from ϕ_1 to ϕ_2 on pf increases from $\cos\phi_1$ to $\cos\phi_2$. Due to increase both in I_a and pf. power generated by the armature increases to meet the increase load.

Over Excitation



When running on light load α_1 is small but I_{a_1} is comparatively larger & leads V by a larger angle ϕ_1.

→ The armature current increases by producing the necessary ~~torque~~ armature power to ~~meet~~ meet the increased applied load.

Summary

i) As load on the motor increases I_a increases regardless of excitation.

ii) For under and over excitation, pf tends to approach unity with increase in load.

iii) Both with under- and over excitation, change in pf is greater than in I_A with increase in load.

DIFFERENT TORQUES OF A SYNCHRONOUS MOTOR

Starting Torque → It is the torque developed by the motor when full voltage is applied to its Stator winding. It is also called breakaway torque

Running Torque → It is the torque developed by the motor under running conditions.

Pull In Torque → A synchronous motor started as induction motor till it runs 2 to 5% below the synchronous speed. The amount of torque at which the motor will pull into step is called the pull in torque.

Pull out Torque → The maximum torque which the motor can develop without pulling out of step or synchronism is called pull out torque.